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Heat transfer of conjugated Graetz problems with laminar counterflow in double-pass concentric circular heat exchangers

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Abstract

A device of external recycle at the ends of double-pass concentric circular heat exchangers with uniform wall temperature, resulting in substantially improving the heat transfer, has been designed and studied theoretically. The theoretical analysis on heat transfer efficiency improvement has been developed using orthogonal expansion technique in power series. The analytical results are also represented graphically and compared with that in an open conduit (without an impermeable plate inserted and without recycle). Considerable improvement in heat transfer is obtainable by employing the external recycle at both ends with a suitable adjustment of the impermeable-sheet position and recycle ratio, instead of using an open conduit.

Keywords: Heat exchangers; Conjugated Graetz problems; Double-pass operations; Concentric circular tubes; External recycle

1. Introduction

Graetz problem is a well-known process describing laminar heat and mass transfer in a confined conduit [1–4]. Since the early studies of Graetz problem were all carried out with ignoring axial conduction or diffusion, those assumptions are not always valid, particularly for low Prandtl number fluids such as liquid metals. Therefore, extended Graetz problems [5–12] and conjugated Graetz problems [13–19] are referred to deal with heat and mass transfer processes between two or more contiguous phases and coupled mutual conditions at the boundaries.

Applications of the recycle-effect concept in designing separation processes and chemical reactors were widely used in absorption, reaction and separation, such as distillation [20], extraction [21], adsorption [22], mass diffusion [23], thermal diffusion [24], loop reactors [25], air-life reactor [26], draft-tube bubble column [27]. The present developments in double-pass countercurrent-flow heat exchangers of multistream systems are fundamentally different due to the velocity sign change. Heat-transfer efficiency enhancement has been investigated theoretically by using orthogonal expansion techniques [28–32] in terms of an extended power series.

The purposes of the present work are to develop an alternative arrangement with inserting the impermeable barrier in parallel to conduct countercurrent

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Nomenclature

C_p	heat capacity, J/kg K
D	hydraulic radius, m
F_m	eigenfunction associated with eigenvalue λ_m
Gz	Graetz number, $4V/\alpha\pi L$
G_m	function defined during the use of orthogonal expansion method
$\bar{h}$	average heat transfer coefficient, kW/m^2 K
h_f	friction loss in conduit, m^2/s^2
I_h	heat transfer improvement, defined by Eq. (12)
I_p	power consumption increment, defined by Eq. (14)
k	thermal conductivity of the fluid, kW/m K
L	conduit length, m
M	reflux ratio, reverse volume flow rate divided by input volume flow rate
$\overline{Nu}$	Nusselt number
P	hydraulic dissipated energy, hp
$2R$	inside diameter of the outer tube, m
Re	Reynolds number
S_m	expansion coefficient associated with eigenvalue λ_m
T	temperature of fluid, K
V	input volume flow rate of conduit, m^3/s
v	velocity distribution of fluid, m/s
r	radial coordinate, m
z	axial coordinate, m

Greek symbols

α	thermal diffusivity of fluid, m^2/s
ξ	longitudinal coordinate, z/L
η	transversal coordinate, r/R
θ	dimensionless temperature, $(T - T_i)/(T_w - T_i)$
κ	channel thickness ratio
λ_m	eigenvalue
μ	fluid viscosity, kg/ms
ρ	fluid density, kg/m^3
ψ	dimensionless temperature, $(T - T_w)/(T_i - T_w)$

Subscripts

a	inner channel
b	annulus channel
F	at the outlet of a double-pass device
i	at the inlet
L	at the end of conduit, $\xi = 1$
0	in a single-pass device without recycle
w	at the wall surface

double-pass concentric heat exchanger device with external recycle at both ends and to formulate the mathematical statement for obtaining an analytical solution to the present conjugated Graetz problem. The present study also discusses the improvement in heat transfer efficiency of such countercurrent double-pass devices and the influence of the impermeable-sheet position on device performance.

2. Temperature profiles

Consider the heat transfer in two channels with thickness $2\kappa R$ and $2(1-\kappa)R$, respectively, which is to divide a circular tube with length L, inside diameter $2R$ by inserting an impermeable sheet with negligible thickness δ ($\ll 2R$) and thermal resistance, as shown in Fig. 1. Before entering the inner tube, the fluid with volumetric flow rate V and the inlet temperature T_i will mix with the fluid of volumetric flow rate MV exiting from the annulus. Counter-current flow is achieved with the aid of conventional pump situated at the end of the annulus and the flow rate then may be regulated and the fluid is completely mixed at the inlet and outlet of the tube.

By following the same mathematical treatment performed in the previous works [33], except the type of recycle, the outlet temperature for double-pass devices (θ_F) as well as for single-pass devices ($\theta_{0,F}$) were also obtained in terms of the Graetz number (Gz), eigenvalues (λ_m and $\lambda_{0,m}$), expansion coefficients ($S_{a,m}$, $S_{b,m}$ and $S_{0,m}$), impermeable-sheet location (κ) and eigenfunctions ($F_{a,m}(\eta_a)$, $F_{b,m}(\eta_b)$ and $F_{0,m}(\eta_0)$). The eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_m, \ldots$) calculated from the following equations:

$$\frac{F'_{a,m}(\kappa)}{F_{a,m}(\kappa)} = \frac{F'_{b,m}(\kappa)}{F_{b,m}(\kappa)} \tag{1}$$

with the orthogonality condition is introduced as follows:

$$\int_0^{\kappa} \left[\frac{v_a \cdot R^2}{L \cdot \alpha}\right] S_{a,m} S_{a,n} \eta F_{a,m} F_{a,n} \, d\eta + \int_{\kappa}^{1} \left[\frac{v_b \cdot R^2}{L \cdot \alpha}\right] S_{b,m} S_{b,n} \eta F_{b,m} F_{b,n} \, d\eta = 0 \tag{2}$$

The dimensionless outlet temperature at $\xi = 1$ is readily obtained from the following overall energy balance in the outer tube

$$\theta_F = 1 - \psi_F = \int_0^1 \frac{\alpha 2\pi L}{V} \left(-\frac{\partial \psi_{b,m}(1,\xi)}{\partial \eta}\right) d\xi = \frac{8}{Gz} \sum_{m=0}^{\infty} \frac{S_{b,m} F'_{b,m}(1)}{\lambda_m} (e^{-\lambda_m} - 1) \tag{3}$$

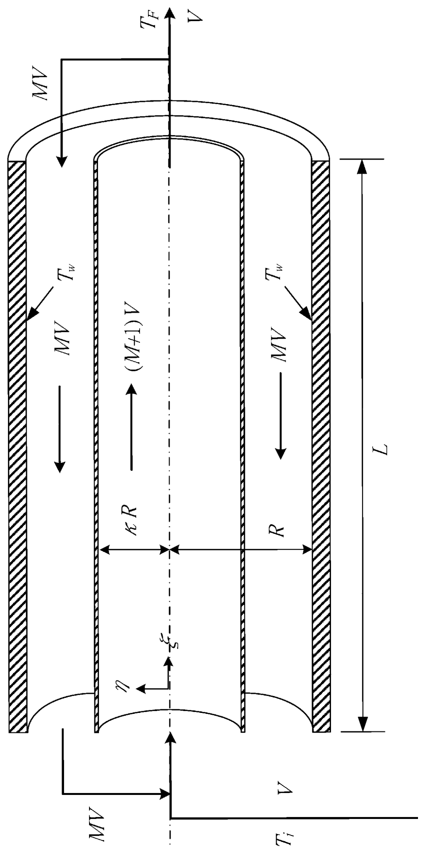

Fig. 1. Schematic diagrams of concentric circular heat exchangers with external recycle at both ends.

in Eq. (3) the left-hand side refers to the net outlet energy while the right-hand side is the total amount of heat transfer from the hot plates into the fluid. The dimensionless outlet temperature for the double-pass devices may also be calculated as follows:

$$\psi_F = \frac{\int_0^{\kappa} v_a 2\pi R^2 \eta \psi_a(\eta,1)\,\mathrm{d}\eta}{V(M+1)} = \frac{2\pi\alpha L}{V(M+1)} \sum_{m=0}^{\infty} \frac{S_{a,m}}{\lambda_m}\left(\int_0^{\kappa} (F''_{a,m}\eta + F'_{a,m})\,\mathrm{d}\eta\right) = \frac{8}{Gz(M+1)} \sum_{m=0}^{\infty} \frac{S_{a,m}}{\lambda_m} \cdot \kappa \cdot F'_{a,m}(\kappa) \tag{4}$$

or

$$\psi_F = -\frac{\int_{\kappa}^{1} v_b 2\pi R^2 \eta \psi_b(\eta,1)\,\mathrm{d}\eta}{MV} = -\frac{2\pi\alpha L}{MV} \sum_{m=0}^{\infty} \frac{S_{b,m}}{\lambda_m}\left(\int_{\kappa}^{1} (F'_{b,m}\eta + F'_{b,m})\,\mathrm{d}\eta\right) = -\frac{8}{GzM} \sum_{m=0}^{\infty} \frac{S_{b,m}}{\lambda_m} \cdot \left[F'_{b,m}(1) - \kappa \cdot F'_{b,m}(\kappa)\right] \tag{5}$$

Also, the dimensionless outlet temperature for the single-pass devices is calculated by

$$\psi_{0,F} = \frac{2\pi\alpha L}{V} \sum_{m=0}^{\infty} \frac{S_{0,m}}{\lambda_{0,m}}\left(\int_0^1 (F''_{0,m}\eta + F'_{0,m})\,\mathrm{d}\eta\right) = \frac{8}{Gz} \sum_{m=0}^{\infty} \frac{S_{0,m}}{\lambda_{0,m}} F'_{0,m}(1) \tag{6}$$

and may be examined using Eq. (7), which is readily obtained from the following overall energy balance in the outer tube

$$\theta_{0,F} = 1 - \psi_{0,F} = \int_0^1 \frac{\alpha 2\pi L}{V}\left(-\frac{\partial \psi_{0,m}(1,\xi)}{\partial \eta}\right)\mathrm{d}\xi = \frac{8}{Gz} \sum_{m=0}^{\infty} \frac{S_{0,m}F'_{0,m}(1)}{\lambda_{0,m}} (\mathrm{e}^{-\lambda_{0,m}} - 1) \tag{7}$$

In obtaining above results, the velocity profiles in present double-pass devices, as shown in Eqs. (3)–(5), were modified from those in the previous works since the type of recycle are different while the mathematical analysis is the same.

$$v_a(\eta) = \frac{2(M+1)V}{\pi(\kappa R)^2}\left(1 - \left(\frac{\eta}{\kappa}\right)^2\right) \quad 0 \leqq \eta \leqq \kappa \tag{8}$$

$$v_b(\eta) = -\frac{2MV}{\pi R^2 - \pi(\kappa R)^2} \frac{\left[1 - (\eta)^2 + \left(\frac{1-\kappa^2}{\ln 1/\kappa}\right)\ln \eta\right]}{\left[\frac{1-\kappa^4}{1-\kappa^2} - \frac{1-\kappa^2}{\ln\frac{1}{\kappa}}\right]} \quad \kappa \leqq \eta \leqq 1 \tag{9}$$

3. Improvement of transfer efficiency

By following the same mathematical treatment performed in the previous work [33], except the type of recycle, the Nusselt number for a double-pass device with recycle may be obtained as follows:

$$\overline{Nu} = \frac{\bar{h}D}{k} = \frac{V}{\pi\alpha L}(1 - \psi_F) = \frac{1}{4}Gz(1 - \psi_F) = \frac{1}{4}Gz\theta_F \tag{10}$$

Similarly, for a single-flow operation without recycle

$$\overline{Nu_0} = \frac{\overline{h_0}D}{k} = \frac{V}{\pi\alpha L}(1 - \psi_{0,F}) = \frac{1}{4}Gz(1 - \psi_{0,F}) = \frac{1}{4}Gz\theta_{0,F} \tag{11}$$

The performance improvement employing a double-pass operation with recycle is best illustrated by calculating the percentage increase in heat-transfer rate, based on the heat transfer of a single-pass operation with the same working dimension and operating conditions, but without impermeable sheet and recycling, as

$$I_{\mathrm{h}} = \frac{\overline{Nu} - \overline{Nu_0}}{Nu_0} = \frac{\psi_{0,F} - \psi_F}{1 - \psi_{0,F}} = \frac{\theta_F - \theta_{0,F}}{\theta_{0,F}} \tag{12}$$

4. Results and discussions

The calculation methods and procedure are exactly the same as those in the previous work [33] and the results thus obtained will be discussed. The changes of mixed inlet temperature $\theta_a(\eta, 0)$ and outlet temperature θ_F with the recycle ratio M and Graetz number Gz as well as the barrier position κ, are in the same tendency as those in the previous work [33]. Both $\theta_a(\eta, 0)$ and θ_F increase with M but decrease as Gz increases, and the $\theta_a(\eta, 0)$ increases with M but decreases as κ moves away from 0.5, especially for $\kappa > 0.5$.

Fig. 2 shows the influence of the recycle ratio M, the channel thickness ratio κ, and the Graetz number Gz on the average Nusselt number $\overline{Nu}$ and $\overline{Nu_0}$. The average Nusselt number $\overline{Nu}$ increases with the recycle ratio, as concluded from Fig. 2. Fig. 2 shows that $\overline{Nu}$ decreases as the κ deviates from 0.5, especially for $\kappa > 0.5$. The reason why $\kappa < 0.5$ is better than $\kappa > 0.5$, for obtaining higher transfer coefficient, is that the heat transfer in the annulus is more effective than that in the inner channel due to the larger temperature gradient.

The Nusselt numbers, $\overline{Nu}$ and $\overline{Nu_0}$, as well as the heat transfer coefficients, $\bar{h}$ and $\overline{h_0}$, can be calculated from Eqs. (10) and (11), respectively, for a double-pass device with recycle and single-pass device without recycle. Fig. 2 gives the graphical representations of $\overline{Nu}$ and $\overline{Nu_0}$ vs. Gz. It is seen from this figure that both $\overline{Nu}$ and $\overline{Nu_0}$ increase with Gz because $\bar{h}$ and $\overline{h_0}$ will be enhanced as the fluid velocity V increases or the length of flow channel L decreases. Moreover, $\overline{Nu}$ is much larger than $\overline{Nu_0}$, except for very small Gz, say $Gz < 10$.

The comparison of $\overline{Nu}$ obtained in the present and the previous studies of double-pass devices with recycle, is indicated in Fig. 3. The values of $\bar{h}$ in the present device are higher, as reasonably inferred from Fig. 3, since the annulus of present device is employed for heating the recycle fluid only while that of the previous one is

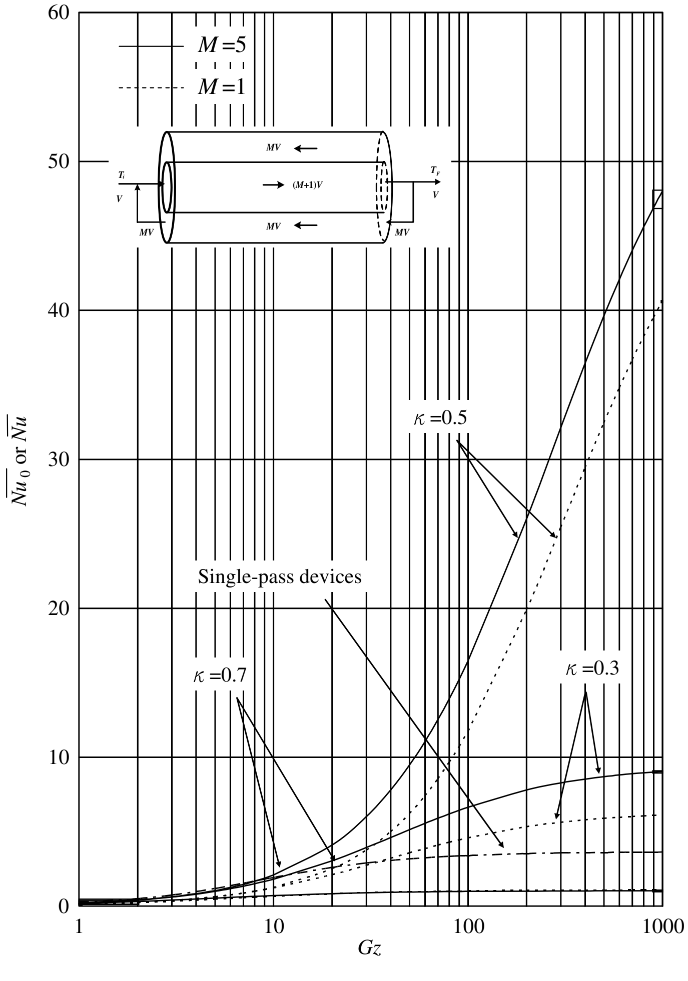

Fig. 2. Average Nusselt number vs. Gz with the channel thickness ratio as a parameter; $M = 1$ and 5.

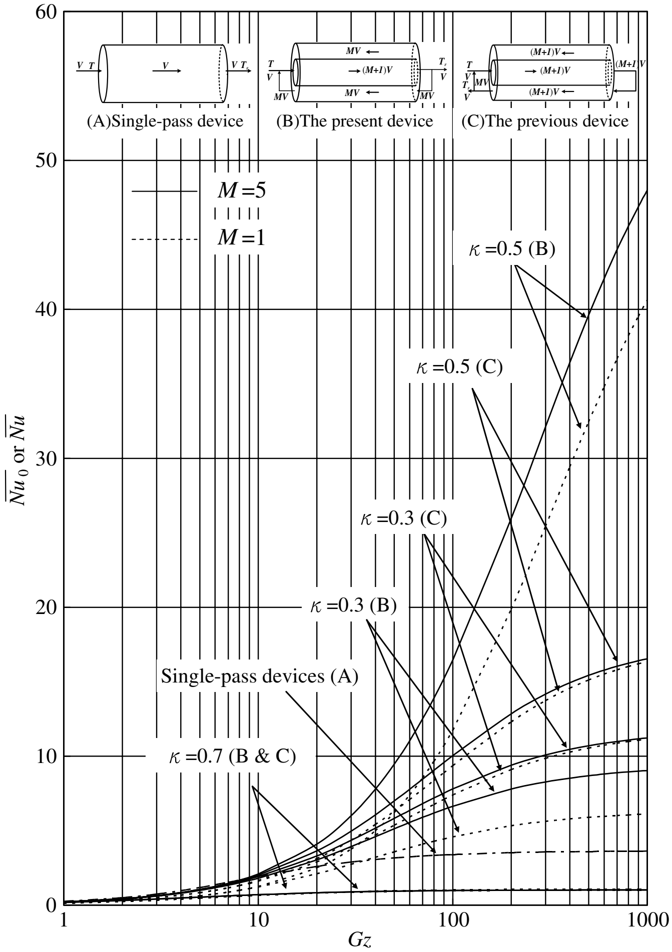

Fig. 3. Average Nusselt number vs. Gz with the recycle ratio and channel thickness ratio as parameters; $M = 1$ and 5.

provided for heating the whole fluid. Further, $\overline{Nu}$ obtained in both devices increase with M, however, the increase in the present device is rather sensitive. This is because the small amount of flowing fluid, say MV, in the present device is heated and the extent of further improvement in transfer efficiency by recycle is rather limited due to the undesirable effect of heat-transfer driving force decrement (temperature difference) while in the previous device [33], the desirable effect of the forced convection increment cannot compensate for the heating larger amount of flowing fluid through the annulus.

Finally, the improvement of device performance I_h was calculated from Eq. (12) theoretically and the results were given in Table 1. It is found from Table 1 that the heat-transfer efficiency improvement of a double-pass device with recycle, based on that of a single-pass device without recycle of the same working dimensions and fluid velocity, increases with the Graetz number and recycle ratio, but decreases with channel thickness ratio κ going away form 0.5, especially for $\kappa > 0.5$. It should be mentioned that the improvement turns to be negative when the Graetz number is relatively small, as also shown by the negative signs in Table 1. In this case, a single-pass heat exchanger without recycle is preferred rather than using the double-pass device even with recycle.

If the laminar flow in concentric tube is assumed, the power consumption increment, I_p, due to the friction losses ($h_{f,a}$ and $h_{f,b}$ for the double-pass devices while $h_{f,0}$ for the single-pass device) in the conduits can be readily defined as

$$I_p = \frac{P - P_0}{P_0} = \frac{V\rho\lfloor (M+1)h_{f,a} + Mh_{f,b} \rfloor - V\rho h_{f,0}}{V\rho h_{f,0}} \tag{13}$$

$$= \frac{(M+1)^2}{\kappa^4} + \frac{M^2}{(1-\kappa^2)(1-\kappa)^2} - 1 \tag{14}$$

The power consumption of a single-pass device will be illustrated using working dimensions as follows: $L = 1.2$ m, $R = 0.2$ m, $V = 1 \times 10^{-4}$ m^3/s, $\mu =$

Table 1
The heat-transfer efficiency improvement with recycle ratio and channel thickness ratio as parameters

I_h (%)	$M = 0.5$			$M = 1$			$M = 3$			$M = 5$		
	κ			κ			κ			κ		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
$Gz = 1$	−65.9	−66.7	−66.7	−49.3	−50.0	−50.3	−23.7	−25.0	−30.8	−17.5	−16.7	−26.3
10	−56.7	−56.6	−59.3	−36.9	−34.9	−66.1	−9.92	−2.41	−63.7	−6.09	8.31	−63.1
100	−6.17	142	−28.9	34.9	245	−69.9	96.5	356	−70.9	95.3	384	−71.1
1000	14.6	879	−19.1	68.3	1019	−70.2	155.3	1176	−71.6	148.9	1222	−71.8

Table 2
The power consumption increment with recycle ratio and channel thickness ratio as parameters

M	I_p		
	$\kappa = 0.3$	$\kappa = 0.5$	$\kappa = 0.7$
0.5	277	36	13
1.0	495	68	37
3.0	1994	303	261
5.0	4499	708	693

8.94×10^{-4} kg/m s, $\rho = 997.08$ kg/m^3. From these values, the friction loss in an open conduit was calculated as follows:

$$P_0 = V\rho h_{f,0} = 1.71 \times 10^{-8}\ \text{W} = 2.29 \times 10^{-11}\ \text{hp} \qquad (15)$$

Though the power consumption increment does not depend on the Graetz number, but it increases as κ moves away from 0.5. It is readily obtained from Eq. (14) that I_p increases with recycle ratio as well as with κ moving away from 0.5, and the results for I_p are represented in Table 2. The theoretical prediction of power consumption estimated by Eq. (14) under these design and operating parameters, $P = 1.03 \times 10^{-7}$ hp, is still small even for $\kappa = 0.3$ and $M = 5.0$. Therefore, the power consumption of double-pass operations with external recycle may be neglected.

5. Conclusion

The influences of recycle effect on the heat transfer efficiency in the double-pass circular tubes with inserting an impermeable tube of negligible thermal resistance were investigated with the recycle ratio and channel thickness ratio as parameters. The method for increasing the outlet temperature and hence Nusselt number in a heat exchanger is either to increase the residence time or to strengthen the premixing effect. Actually, the introduction of recycle to the heat transfer devices comes out with two conflicting effects: the desirable effect of forced convection coefficient increment and undesirable effect of heat-transfer driving force decrement (temperature gradient). However, the external recycle still have positive influences on the outlet temperature for large Gz. This is because larger amount of recycle fluid (larger M) for the flowing fluid through annulus will enlarge temperature-gradient increment, then the extent of further improvement in transfer efficiency by recycle effect is achieved.

It is obvious that the mathematical treatments developed in this study to concentric circular heat exchangers can also be applied to the conjugated Graetz problems in heat- or mass-transfer devices with constant heat flux or mass flux on the boundary if the boundary conditions of the interaction between streams or phases are suitably changed.

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